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Object-Oriented Optimization of Discrete Structures

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A new object-oriented method is proposed for the optimum design of discrete structures being subjected to various local and global constraints. The method uses an element-based information and procedure instead of the global ones, such as the Lagrange multiplier. The principle of the method is that each element is able to evaluate its resource margins with respect to various local and global constraints, and the element reduces its resource based on the minimum of those resource margins. This simple procedure is used iteratively to obtain an optimum solution. It is found from several numerical experiments that the proposed method is effective for the optimum design of discrete structures, such as trusses.

Nomenclature

A = sectional area
G_1, G_2 = constraints
P = member force
P_{cr} = buckling load
P_0 = initial design point
V = volume of truss member
σ = stress of truss member
σ_t = tensile stress

Subscripts and Superscripts

b = buckling
cr = buckling load
i = member index
(k) = kth iteration cycle
m = margin
t = tensile

Introduction

KNOWLEDGE-BASED systems work with knowledge bases not with procedures written in the computer programs. An object-oriented approach is a typical method for constructing knowledge bases. Knowledge-based systems yield remarkable benefits in modeling and simulation of engineering systems, but their use in the optimization field of structures is very limited. A knowledge-based optimization method is different from the conventional mathematical programming method. Knowledge comprising rules, principles, and policies is stored in a knowledge base, and each element of a discrete structure changes itself using the knowledge base. This approach provides remarkable capabilities in treating complicated systems and verifying knowledge for optimization.

Much research on the development of knowledge-based systems in the structural engineering field has been conducted.[1–3] An object-oriented approach also has been introduced in some cases.[4] These systems, however, are not operated in a completely object-oriented manner and are not intended for structural optimization. The author has proposed a new object-oriented method of modeling and analysis of truss structures.[5] In this paper, a new object-oriented optimization method is proposed. The emphasis here is placed on the proposal of a new concept for the optimization of discrete structures, not on implementation of the algorithm with an object-oriented language, although it is very simple and can be combined with other knowledge in a knowledge base.

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Object-Oriented Approach to Optimization

The concept of object-oriented analysis and design is introduced in the computer-aided software engineering field. The object-oriented analyst views the world as objects with data structures and methods and events, which trigger operations that change the state of objects.[6] This approach qualitatively enhances the design, creation, and maintenance of the software system. A great deal of this added power is derived from modularity.

The object-oriented approach provides not only the software benefit, but also the change in our perspectives on all systems. Miki and Murotsu[5] proposed a new object-oriented method for the geometric and structural analysis of truss structures. The main components in the analysis are the truss members and truss nodes. They contain individual knowledge, which are very simple and essential. The most important knowledge of the truss nodes is that a node moves if it has nonzero resultant force with respect to the connecting truss members, a supporting force, and an external force, if any. The most important knowledge about a truss member is the response to its member force obtained from the strain calculated from the coordinates of the end nodes.

Such knowledge is essential, and it works without any problems for complicated behaviors such as any geometric, structural, and material nonlinearities. It is remarkably effective in object-oriented structural analysis.

Object-oriented optimization is a new concept, and it is an element-based optimization where each element changes its design variables using its knowledge base. The most important point is that the information for changing the design variables should be as local as possible for two reasons: 1) the acquisition of information about all the elements is expensive and 2) we want to discover another resizing rule, which is different from some resizing rules proposed so far based on the Kuhn–Tucker conditions. The change in the design variables in each element includes not only the size variables, such as the sectional area, but also the shape variables, such as the sectional shape. For example, a truss member reshapes its sectional shape from a solid circle to a hollow circle to increase its buckling strength using its knowledge base. The reason why we call the proposed method an object-oriented and knowledge-based optimization is that the proposed method focuses on an element-based algorithm, the modularized knowledge of the elements of a system, and the interaction of this knowledge. The final goal of our research is to establish a distributed optimization method. The comparison between the proposed method and the conventional optimality criteria methods is discussed later.

Optimization of Discrete Systems

This paper deals with the optimization of discrete systems. A discrete system consists of many elements which have their own states and behaviors. The relations between the elements generally are very complicated. The system requires a certain resource, and the resource is distributed to all of the elements according to their requirements. The system has both local and global constraints.

The objective of the optimization of such system is to minimize the resource being subjected to those constraints. Many optimization problems can be converted to this type of resource optimally distribution problem.

The optimization method proposed here is as follows.

1) Each element can evaluate its resource margins with respect to the given local constraints.

2) Each element can evaluate its resource margins with respect to the given global constraints. This process is performed by evaluating the global response.

3) The minimum value of these resource margins is the critical resource margin in the element.

4) Each element reduces the critical resource margin, and the system attains another equilibrium state with less resource. This process is called the resource reduction process.

5) The critical resource margins of all of the elements becomes zero after an appropriate repetition of the listed procedures.

6) Among two arbitrary elements, a certain amount of resource is transferred from one to another if it yields some increase in the critical resource margin, that is, the transfer is effective. The element which obtains a positive resource margin reduces it. This process is called the resource transfer and reduction process.

7) Repeat the resource transfer and reduction.

The resource margin is calculated by evaluating the sensitivity of the constraint to the resource. The sensitivity can be evaluated by giving a small amount of resource to the element. In numerical optimization methods, these procedures are expressed by mathematical expressions and a sophisticated optimization method, such as conjugate gradient methods, is introduced. Such an approach, however, is not effective for very complicated problems where the response is highly nonlinear, the objective function as well as constraint functions are not well behaved, the system exhibits mode changes, the elements have high nonlinear relations between each other, and so on. The proposed object-oriented optimization method, however, does not contain complicated mathematical procedures, rather simple rules or principles. We can try a lot of policies to optimize a system, and we can discover what is happening and what is important during optimization.

The optimization method proposed here is first introduced for solving the following simple linear optimization problem.

Minimize $f = (1/2)x_1 + x_2$

subject to

$$g_1 = -x_1 - 3x_2 + 3 \leq 0, \qquad g_2 = -x_1 - x_2 + 2 \leq 0$$

$$x_1 \geq 0, \qquad x_2 \geq 0 \tag{1}$$

This problem can be rewritten to a minimum resource problem by replacing variables as follows:

$$R_1 = \tfrac{1}{2}x_1, \qquad R_2 = x_2 \tag{2}$$

Then the original problem becomes as follows.

Minimize $R = R_1 + R_2$

subject to

$$G_1 = -2R_1 - 3R_2 + 3 \leq 0, \quad G_2 = -2R_1 - R_2 + 2 \leq 0$$

$$R_1 \geq 0, \qquad R_2 \geq 0 \tag{3}$$

where R is a resource.

Figure 1 shows the history of the solution during the proposed optimization method. P_0, the initial design point is in the feasible region, but it can be in the infeasible region. The resource margins of R_1 are M_0 for the positive constraint, M_1 for constraint G_1, and M_2 for constraint G_2. The minimum resource margin is M_2, and it is discarded so that the design point is moved to P_1.

The critical resource margins for R_1 and R_2 are both zeros at this point, then element 2 transfers a certain amount of resource to element 1, and the design point moves to P_2. The resource transfer from element 1 to element 2 is not effective since it does not yield an increase in their critical resource margins.

At P_2, both elements have positive critical resource margins, and element 2 reduces its critical resource margin. The design point becomes P_3. After repeating such procedures, the optimum solution is obtained as the intersecting point of lines G_1 and G_2.

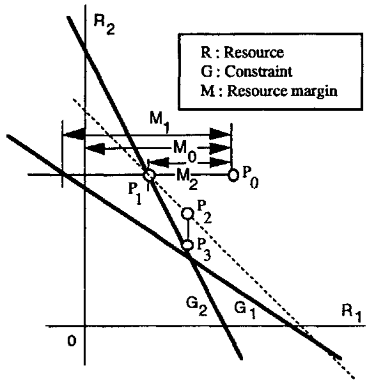

Fig. 1 Concept of the object-oriented optimization for a simple linear optimization problem.

Optimization of Truss Structures

The principle mentioned in the preceding section is rewritten for truss structures. The problem is to obtain the optimum sectional areas of the truss members of the minimum volume truss under the given constraints. The local constraints are that a truss member does not fail in tension and does not buckle in compression. The global constraints are imposed to the displacements of the truss nodes.

The object-oriented optimization method is as follows.

1) Calculate the volume margins of a member with respect to the tensile strength, compressive buckling, and the constraint on a node displacement.

2) Obtain the minimum value of the volume margins, which is the critical volume margin of the member with respect to all of the constraints.

3) Reduce the volume of the member by the critical volume margin, and perform structural analysis.

4) Repeat until all of the critical volume margins of the member become approximately zero.

5) Transfer a small amount volume between two members, and evaluate the critical volume margins of the two members.

6) If they are positive, then they are reduced, otherwise the volume transfer is canceled.

7) Repeat the volume transfer and reduction until the optimum design is obtained.

The whole knowledge base is written in an object-oriented programming language, Smalltalk, which is a typical object-oriented programming language. However, using such object-oriented language is not essential since the concept of the object-oriented optimization lies the object-oriented algorithm instead of the object-oriented language.

The most important knowledge is the procedure to obtain volume margins with respect to various constraints. The volume margin for tensile strength is represented by the following expression although it is not used explicitly in the knowledge base:

$$V_{mi}^{t,(k)} = V_i^{(k)}\left[1 - \left(\sigma_i^{(k)}/\sigma_t\right)\right] \tag{4}$$

The volume margin for buckling is represented by the following expression, although it is not used explicitly in the knowledge base:

$$V_{mi}^{b,(k)} = V_i^{(k)}\left(1 - \sqrt{P_i^{(k)}/P_{\mathrm{cr}}^{(k)}}\right) \tag{5}$$

The square root is very important since our object-oriented approach attaches importance to the physical entity. Equation (5) is rewritten as follows:

$$V_i^{b,(k+1)} = V_i^{b,(k)} - V_{mi}^{b,(k)} = V_i^{b,(k)}\sqrt{P_i^{(k)}/P_{\mathrm{cr}}^{(k)}} \tag{6}$$

Consequently,

$$V_i^{b,(k+1)}/V_i^{b,(k)} = \sqrt{P_i^{(k)}/P_{\mathrm{cr}}^{(k)}} \tag{7}$$

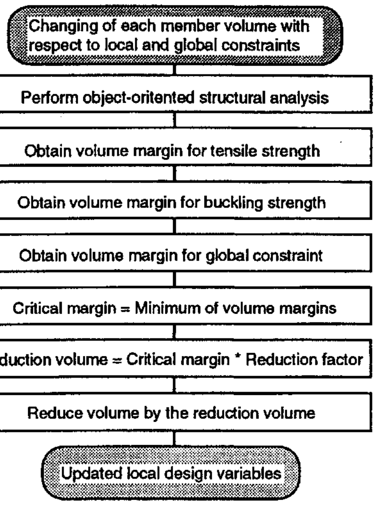

Fig. 2 Method to change the volume of a truss member.

Since the length of the member is constant, Eq. (7) becomes Eq. (8),

$$\left(\frac{A_i^{b,(k+1)}}{A_i^{b,(k)}}\right)^2 = \left(\frac{r_i^{(k+1)}}{r_i^{(k)}}\right)^4 = \frac{P_i^{(k)}}{P_{\rm cr}^{(k)}} \tag{8}$$

where the sectional shape of the member is assumed to be a circle and r is the radius of the circle. Multiplying both sides of Eq. (8) by $(\pi^3 E/4l^2)$, we obtain the following equation:

$$P_{\rm cr}^{(k+1)} = P_i^k \tag{9}$$

That is, the buckling strength of member i at $k+1$ cycles becomes the member force at k cycles. The member will change its sectional area to support the present member load. Therefore, the physical meaning is consistent. If another power instead of 1/2 is used in Eq. (5), then the convergence of the solution is not assured.

The volume margin for a global constraint can be represented by the following expression:

$$V_{mi}^{G_j,(k)} = \alpha\left[G_j^{(k)} \Big/ \left(\frac{\partial G_j^{(k)}}{\partial V_i}\right)\right] \tag{10}$$

where G_j is the jth global constraint.

We introduce a responsibility factor α in evaluating the volume margin for a global constraints, as shown in Eq. (10). The factor represents the degree of contribution for the member to the global constraint. When α is unity, only the member is fully responsible for the satisfaction of the global constraint, whereas an α of zero means that the member has no responsibility for it. The responsibility factor is determined heuristically in order to obtain a good convergence. A value of $1/N$ (where N is the number of the elements) is a reasonable one.

It should be noted that the sensitivity is obtained in an object-oriented way. A small amount of volume is actually added to a member, and the difference of the global constraint function is evaluated. Since an object truss cannot be copied like an actual truss, the truss itself changes slightly during the evaluation process of the sensitivity.

The method for changing the volume with respect to the given local and global constraints is shown in Fig. 2. For simplicity, the number of global constraints is one in this case, but we can treat a number of global constraints.

In the method shown in Fig. 2, several volume margins for various constraints are obtained, then the minimum margin is discarded. We introduce a reduction factor to calculate the discarding volume. The reduction factor represents the degree of the reduction of the critical resource margin. When the factor is unity, the member reduces all of the critical resource margin, while the value of the factor is reduced in order to get prudence. This factor being less than unity is effective for the stable convergence. The reduction factor is determined heuristically, and the factor of 0.5 is used in this paper.

Results and Discussions

Three-Member Truss

The object-oriented optimization method is found to be effective for linear optimization problems, as mentioned before. We use this method for a nonlinear optimization problem here.

An indeterminate truss structure with three members is shown in Fig. 3. The problem is to obtain the minimum volume structure being subjected to local and global constraints. The design variables are the sectional areas of the members. The local constraints are tensile and buckling strengths, and the global constraint is imposed on the x coordinate of node 4, as follows:

$$\begin{array}{lll} \sigma_i \le \sigma_t, & P_i \ge P_{{\rm cr},i} & (i = 1, 2, 3) \\ x_j \le x_{j,0} & (j = 4, & x_{j,0} = 0.53) \end{array} \tag{11}$$

where σ_i is the stress of the ith member, P_i the axial force of the ith member, and x_j the x coordinate of the jth node.

The material of the member is a flexible polymer which has Young's modulus of 0.1 GPa and tensile strength of 40 MPa. The sectional shape of the member is a circle with radius of 10 mm as an initial value. The magnitude and direction of the external force is 1700 N and 30 deg, respectively, and it is applied at node 4.

The history of the total volume of the 3-member truss during the iteration process is shown in Fig. 4. The initial design is not within the feasible region so that the total volume increases rapidly at first. The history of the member force is shown in Fig. 5. From these figures, the feasible design solution is where each member has critical volume margin of approximately zero after three repetitions of the resource reduction process.

However, it is not an optimum solution. We performed 20 cycles of the resource reduction processes, and then performed 20 cycles of the resource transfer and reduction processes. In the resource transfer and reduction processes, the volumes of members 2 and 3 are transferred to member 1 and the resulted volume margin is discarded. This causes the remarkable change in the load path, as shown in Fig. 5. The optimum solution is obtained after about 10 cycles of the resource transfer and reduction processes.

The histories of the stresses of the members and the coordinates of node 4 are shown in Figs. 6 and 7, respectively. It is seen that the

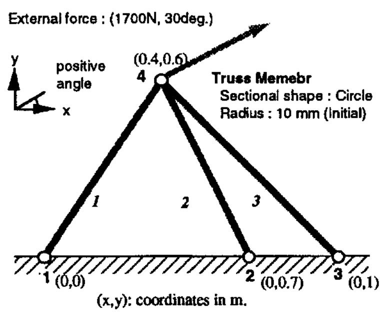

Fig. 3 3-member truss for optimization.

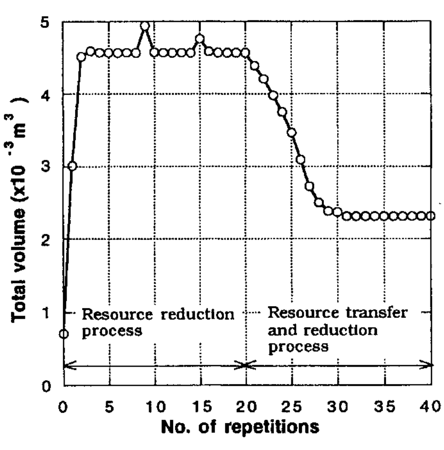

Fig. 4 History of the total volume of the 3-member truss during the repeated optimization processes.

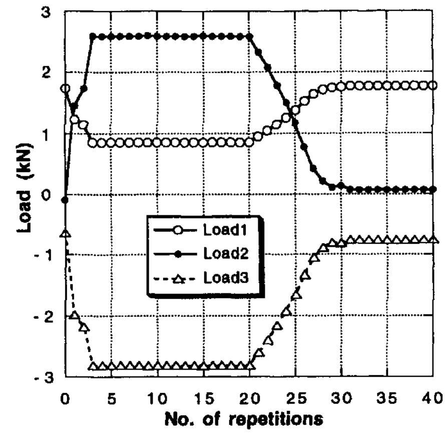

Fig. 5 History of the member forces of the 3-member truss during the repeated optimization processes.

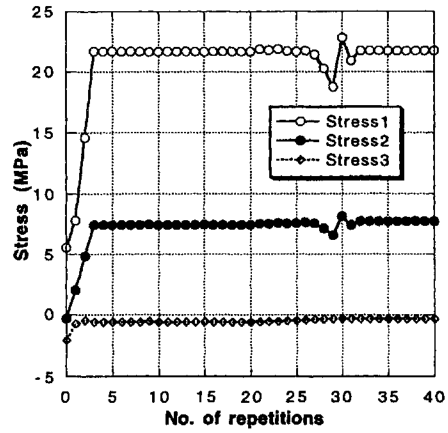

Fig. 6 History of the member stresses of the 3-member truss during the repeated optimization processes.

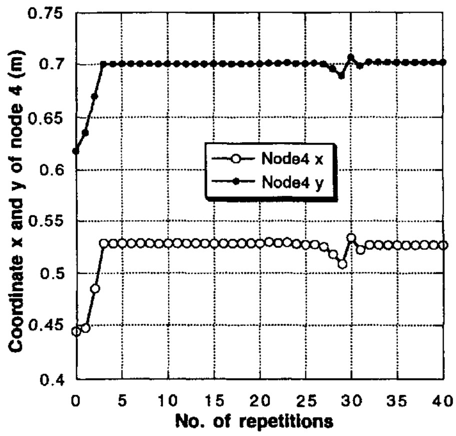

Fig. 7 History of the coordinates of node 4 of the 3-member truss during the repeated optimization processes.

constraint on the tensile strength is not active for all members, and the global constraint is satisfied for almost every cycle.

The optimum solution is shown in Table 1 where member 2 is very thin, and the truss becomes a determinate structure. It is found that the sectional area of member 1 is determined from the global constraint, and the sectional area of member 3 is determined from the buckling constraint. From this result, the proposed optimization method is found to be effective for nonlinear problems with local and global constraints.

Optimization of 11-Member Truss

The effectiveness of the proposed optimization method is investigated for a more complex problem. The problem is to obtain a minimum volume design of an 11-member truss shown in Fig. 8.

Table 1 Result of the minimum volume design of the 3-member truss (after 40 repetitions)

Member	Sectional area, mm^2	Load, N	Stress, MPa	Buckling strength, N
1	81.44	1768.8	21.72	—
2	9.22	71.2	7.73	—
3	2646.85	−763.5	−0.29	−764.2

Total volume = 2.311×10^{-3} m^3.

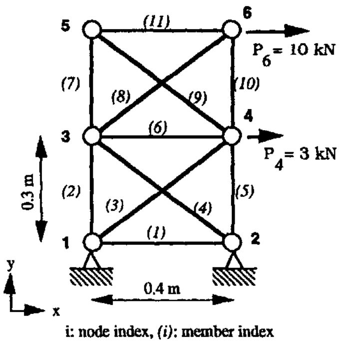

Fig. 8 11-member truss.

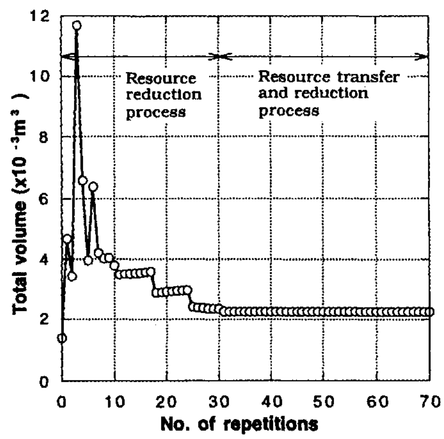

Fig. 9 History of the total volume of the 11-member truss during the repeated optimization processes.

The objective is to minimize the total volume of the truss being subjected to several constraints. The design variables are the sectional areas of the member. The local constraints are tensile and buckling strengths, and the global constraint is imposed on the x coordinate of node 6, as follows:

$$\begin{aligned} \sigma_i &\le \sigma_t, & P_i &\ge P_{\mathrm{cr},i} & (i &= 1, \ldots, 11) \\ x_j &\le x_{j,0} & (j &= 6, & x_{j,0} &= 0.43) \end{aligned} \tag{12}$$

The material of the member is a polymer which has Young's modulus of 1 GPa and tensile strength of 40 MPa. The sectional shape of the member is a circle with radius of 10 mm as an initial value. The magnitudes of the external forces are 10 and 3 kN, and they are applied horizontally at nodes 6 and 4, respectively, as shown in Fig. 8.

For the 11-member truss, we performed 30 cycles of the resource reduction processes and 40 cycles of the resource transfer and reduction processes. The history of the total volume of the truss is shown in Fig. 9. The initial design is not within the feasible region so that the total volume increases at first similarly to the case of the 3-member truss.

The histories of the sectional areas of the members during the resource reduction processes are shown in Fig.10. Some members are vanishing in the first 10 cycles, members 4 and 5 show remarkable change, whereas members 2, 8, and 10 show a smooth change during the resource reduction processes. The coordinates of node 6 are almost constant, although the sectional areas change

Table 2 **Result of the optimum design of the 11-member truss**

Member	Sectional area, mm^2	Load, N	Stress, MPa	Buckling strength, N
1	+0.0	0.0	0.0	—
2	1,090.9	17,081.7	15.66	—
3	0.1	4.4	34.95	—
4	2,298.0	−16,590.0	−7.22	−16,590.5
5	890.4	−6,917.6	−7.77	−6,918.0
6	85.4	3,414.7	40.00	—
7	+0.0	+0.0	1.03	—
8	519.5	12,301.4	23.68	—
9	26.0	−1.9	0.07	−2.1
10	893.3	−6,957.8	−7.79	−6,964.5
11	+0.0	+0.0	1.08	—

Total volume = 2.318×10^{-3} m^3. Note: +0.0 means very small.

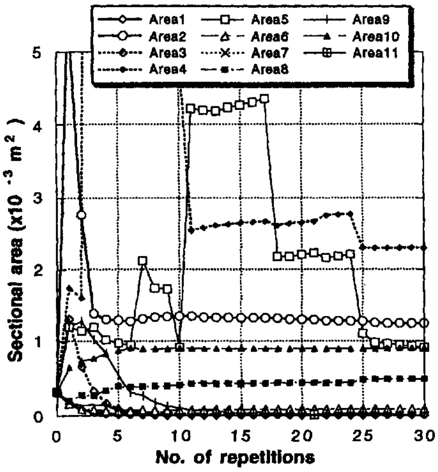

Fig. 10 History of the sectional areas of the members during the repeated optimization processes.

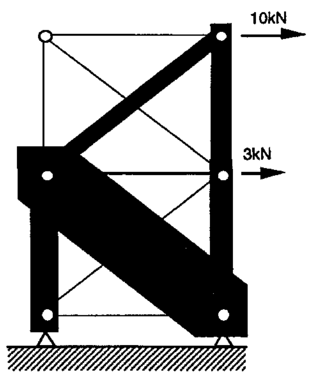

Fig. 11 Distribution of the sectional areas of the members of the optimized truss.

remarkably. The details of optimum design are shown in Table 2, and the distribution of the sectional area of the optimum design is shown schematically in Fig. 11. The main load path is clearly seen.

From Fig. 9, it can be seen that about 25 cycles are required to obtain a feasible design in which each member has the critical volume margin of approximately zero. What is surprising is that the design obtained after 30 cycles of the resource reduction process is approximately an optimum design, and the resource transfer and reduction processes do almost nothing. This is remarkably different from the result of the 3-member truss, which required the resource transfer and reduction processes.

This surprising result means that the resource reduction process is necessary and sufficient to obtain an optimum solution, and the resource transfer and reduction process is not necessary for complex systems. This conclusion provides a great advantage to the proposed optimization method since the resource transfer and reduction process are time consuming.

The reason why the resource reduction process yields an optimum design for complex systems is not clear, but we think that in complex systems, a change in the resource in an element causes a lot of nonlinear effects to other elements, and this disturbance does not allow the elements to stay at nonoptimum design. For a simple system like the 3-member truss, this disturbance is rapidly converged, and a stable nonoptimum design exists during the resource reduction processes.

In other words, a stable nonoptimum design can exist only when all of the critical resource margins are approximately zero at the same time, and such a situation does not occur in complex systems; then a disturbance continuously exists, and it reduces the surplus of the total resource. However, more precise investigation is necessary. This remains for future study.

Discussion

Comparison with Optimality Criteria Method

Venkayya[7] and Khot[8] have proposed some resizing algorithms based on optimality criteria to design minimum weight structures. Those algorithms ensure that the optimum solution will be obtained since they are based on rigorous optimality criteria derived from the Kuhn–Tucker conditions. On the other hand, the proposed resizing rules shown in Eqs. (4), (5), and (10) are not derived from such optimality criteria. In optimality criteria, the gradients of a global constraint with respect to all of the design variables are necessary to estimate the Lagrange multiplier, but the proposed method does not use such global information. Each truss member resizes its own design variables independently of other members. Therefore, the proposed resource reduction algorithm is not an optimality criteria method.

The resource reduction process, therefore, does not ensure the optimum solution, but it ensures the feasible solution with some active constraints. The resource transfer and reduction process ensures the optimum solution since the resource transfer becomes ineffective where all design variables are equally cost effective in changing the constraint. Such a design point is the optimum. That is, the resource transfer and reduction algorithm proposed here can be said to be an optimality criteria method.

Therefore, there is a possibility of obtaining the optimum using only the resource transfer and reduction process. It is very expensive, however, since resource exchange in many elements takes a very long computation time, and it can be used with a feasible initial design. The resource reduction process yields a feasible solution.

Optimization Method Without Resource Transfer

For the 11-member truss, the optimum solution is obtained only by the resource reduction processes, as mentioned before. The resource transfer takes a long time in complex systems since the number of combinations of two arbitrary elements engaging the transfer is very large. Therefore, the result that the resource reduction process is sufficient to optimize complex systems makes the object-oriented optimization method useful.

We think the most important aspect in introducing the object-oriented approach to systems analysis and design is the change in perspective, as well as the software benefit. It is very important to find the individual knowledge or rule of each element in a system in order to make the system optimum. Conventional mathematical programming methods are global methods whereas the proposed method is a local or an element-oriented method.

We call such element-oriented method an intelligent autonomous element method here. This method does not use a global information, such as a global stiffness matrix in the finite element method, or a Lagrangian function which contains an objective function and constraints. Each element has its own knowledge, behaves based on its knowledge and local information (autonomous), and it knows how to behave in order to get the system optimization (intelligent).

Effectiveness of the Proposed Method

The effectiveness of the proposed method is investigated by changing the initial design. A lot of initial designs including very thin members, very thick members, and the worst design, which is contrary to the optimum design obtained, are tested. The optimum

Table 3 Comparison of optimization methods

Method	Sectional area, m^2	Total volume, m^3	Comment
Proposed	m4: 2298.0 m6: 85.37	2.318	Robust (Nonlinear)
SUMT	m4: 2013.0 m6: 1.578	2.192	Not robust (Linear)
GA	m4: 2010.0 m6: 1.0e−9	2.271	Robust (Linear)

SUMT: Sequentially unconstrained minimization technique.
GA: Genetic algorithm (Pop. 30 × 1000 generations)
m4: member 4, m6: member 6.
Linear and nonlinear mean structural analysis methods.

solutions obtained with these initial designs are very similar to the result shown in Table 2. Consequently, the proposed method is found to be effective and robust for initial designs.

The optimum design obtained by the proposed method is compared with those obtained by other optimization methods, as shown in Table 3 where a nonlinear mathematical programming method and a genetic algorithm are used. It should be noted that those optimizations were conducted with a linear finite element analysis, whereas the object-oriented truss analysis proposed in the previous paper is essentially a nonlinear structural analysis. The member forces are generally smaller in the linear analysis. For example, axial forces in members 4 and 6 are −12,577 and 61 N, respectively, in the linear analysis, whereas they are −16,590 and 3,415 N, respectively, in the nonlinear analysis. Therefore, the total volume becomes large using a nonlinear analysis, which is more exact.

The total volumes of the results obtained by the three methods are very similar to each other, and the proposed method is found to be valid. The nonlinear mathematical programming method sometimes causes divergence depending on initial designs.

Conclusions

A new object-oriented optimization method is proposed. The method is developed for the optimum design of discrete systems, such as truss structures under various local and global constraints. The basic principle is that each element of the system behaves on the basis of its knowledge and local information in order to optimize the system. The resource reduction process and the resource transfer and reduction process are developed and investigated.

The proposed method is implemented in a knowledge-based system written in an object-oriented language, and it is found to be valid for optimization of several truss structures. Further, it is found that use of only the resource reduction process is even sufficient for optimization.

In the proposed method, resource margins are evaluated with respect to the respective constraints, and each element changes its resource based on the minimum of those margins. This new algorithm is found to be effective also. Additionally, the exact form for evaluating the volume margin for the buckling of members is shown.

Mathematical programming methods are very sophisticated and useful in well-behaved problems with careful setting of initial designs and various parameters. However, they are very mathematical, and we sometimes ignore the physical aspect of the problem. The object-oriented optimization method proposed here is approached from a contrary direction. We focus on the individual elements of the system and on their behaviors from every possible physical aspect. This approach takes much time to obtain results, but it is useful for the analysis and design of complex systems with highly nonlinear behaviors.

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